Math 10B with Professor Stankova
Worksheet, Discussion #19; Tuesday, 4/3/2018
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1 Partial Fractions

1.1 Concepts

1. Partial fractions allow us to compute an antiderivative of an expression of the form $P(x)/Q(x)$, where P, Q are polynomials, more easily (these are just fractions where the numerator and denominator are both fractions). First long divide so that the degree or highest term of the polynomial P is less than Q. Then factor $Q(x)$ into linear factors if you can, or else quadratic factors. Then for each factor, write the simplification of the form:

Factor	$ax+b$	$(ax+b)^n$	ax^2+bx+c	$(ax^2+bx+c)^n$
Expression	$\frac{A}{ax+b}$	$\frac{A_1}{ax+b}+\frac{A_2}{(ax+b)^2}+\cdots$	$\frac{Ax+B}{ax^2+bx+c}$	$\frac{A_1x+B_1}{ax^2+bx+c}+\frac{A_2x+B_2}{(ax^2+bx+c)^2}+\cdots$

Afterwards, find what these constants are. One good way to do this is to multiply everything by $Q(x)$ to clear denominators and then plug in different values of x.

1.2 Examples

2. Find $\int \frac{x^2}{x^2+3x-18}dx$.

3. Find $\int \frac{x^3+3x^2+3x+3}{(x+1)^2(x^2+1)}dx$.

1.3 Problems

4. True False To find the partial fraction decomposition of $\frac{4x^3}{(x-1)(x+2)^2}$, we set it equal to $\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$ and solve for A, B, C.

5. Integrate $\int \frac{5x}{x^2-9x-36}dx$.

6. Integrate $\int \frac{4x^2}{(x-1)(x-2)^2}dx$.

7. Set up the partial fraction decomposition of $\frac{8x^3+3x^2+1}{(x-1)^2(x^2+4)^2}$ (you don't have to solve for the coefficients).

8. Integrate $\int \frac{\sec^2(x)}{\tan(x)^2-\tan(x)}dx$.

1.4 Extra Problems

9. Integrate $\int \frac{5x+17}{x^2+2x-15} dx$.

10. Integrate $\int \frac{2x^3-12x^2+28x-23}{(x-2)^2(x-1)^2} dx$.

11. Set up the partial fraction decomposition of $\frac{3x^2+1}{(x-1)(x^2+4)^2(x^2+2x+2)^2}$ (you don't have to solve for the coefficients).